Exercise 37

For the following exercises, solve the equations over the complex numbers.

$$x(x-4)=20$$

Solution

Rewrite the equation in standard form.

$$x^2-4x=20$$

$$x^2-4x-20=0$$

The two terms with x, x^2 and $4x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x+B)^2=x^2+2xB+B^2$$

Notice that $2B=-4$, which means $B=-2$ and $B^2=4$. Add and subtract 4 from the left side and apply the identity.

$$(x^2-4x+4)-20-4=0$$

$$(x+(-2))^2-24=0$$

$$(x-2)^2-24=0$$

Now that x appears in only one place, it can be solved for. Add 24 to both sides.

$$(x-2)^2=24$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x-2)^2}&=\sqrt{24}\\&=\sqrt{4(6)}\\&=\sqrt{4}\sqrt{6}\\&=2\sqrt{6}\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x-2$.

$$|x-2|=2\sqrt{6}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x-2=\pm 2\sqrt{6}$$

Add 2 to both sides.

$$x=2\pm 2\sqrt{6}$$

Therefore, $x=\{2-2\sqrt{6},2+2\sqrt{6}\}$.